Equivalence of Deterministic Nested Word to Word Transducers

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(Joint work with Sławek Staworko, Aurelien Lemay, Joachim Niehren)

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University of Lille

Mostrare, 06 May 2009

Motivation

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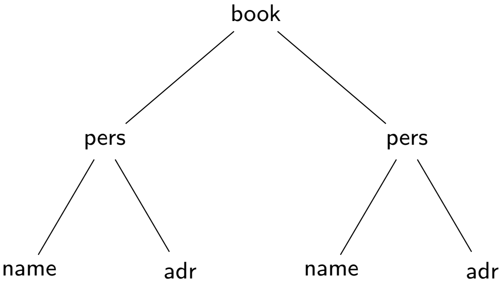
</book>                 ->  </table></body></html>
```

Overview

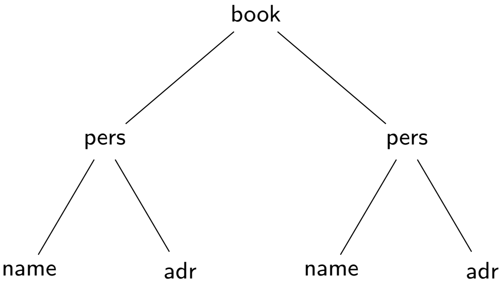
Outline

Nested Word Automata



- Visibly Pushdown Automata [Alur et al'04]
- Streaming Tree Automata [Gauwin et al'08]

Nested Word Automata



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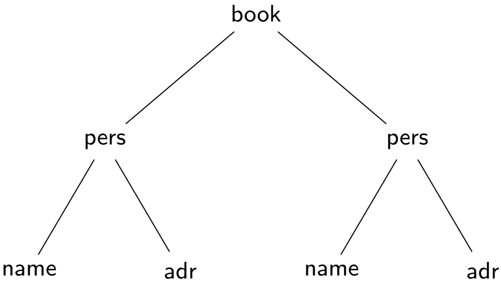
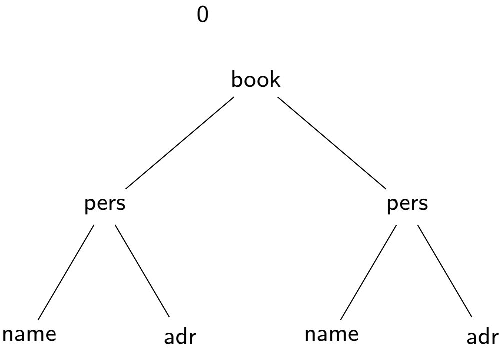
Nested Word Automata



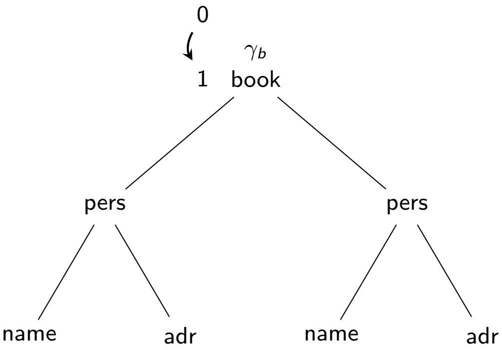
$$0 \xrightarrow{\texttt{op}\ book:\gamma_b} 1$$
$$1 \xrightarrow{\texttt{op}\ pers:\gamma_p} 2$$
$$2 \xrightarrow{\texttt{op}\ name:\gamma_n} 3$$
$$3 \xrightarrow{\texttt{cl}\ name:\gamma_n} 4$$
$$4 \xrightarrow{\texttt{op}\ adr:\gamma_a} 3$$
$$3 \xrightarrow{\texttt{cl}\ adr:\gamma_a} 5$$
$$5 \xrightarrow{\texttt{cl}\ pers:\gamma_p} 1$$
$$1 \xrightarrow{\texttt{cl}\ book:\gamma_b} 6$$

Nested Word Automata



$$
\begin{aligned}
0 &\xrightarrow{\texttt{op}\ \mathit{book}:\gamma_b} 1\\
1 &\xrightarrow{\texttt{op}\ \mathit{pers}:\gamma_p} 2\\
2 &\xrightarrow{\texttt{op}\ \mathit{name}:\gamma_n} 3\\
3 &\xrightarrow{\texttt{cl}\ \mathit{name}:\gamma_n} 4\\
4 &\xrightarrow{\texttt{op}\ \mathit{adr}:\gamma_a} 3\\
3 &\xrightarrow{\texttt{cl}\ \mathit{adr}:\gamma_a} 5\\
5 &\xrightarrow{\texttt{cl}\ \mathit{pers}:\gamma_p} 1\\
1 &\xrightarrow{\texttt{cl}\ \mathit{book}:\gamma_b} 6
\end{aligned}
$$

Nested Word Automata



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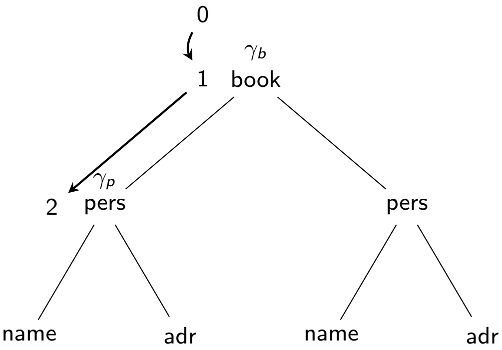
$< book >$

Nested Word Automata



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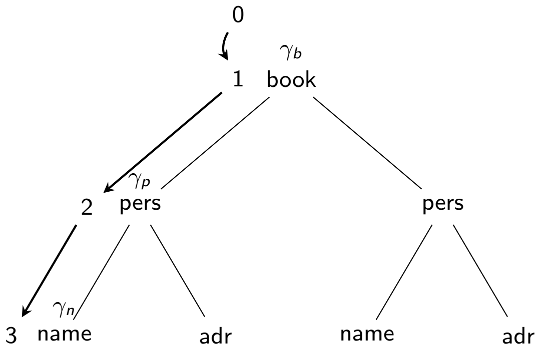
$< book >$

$< pers >$

Nested Word Automata



$$
\begin{aligned}
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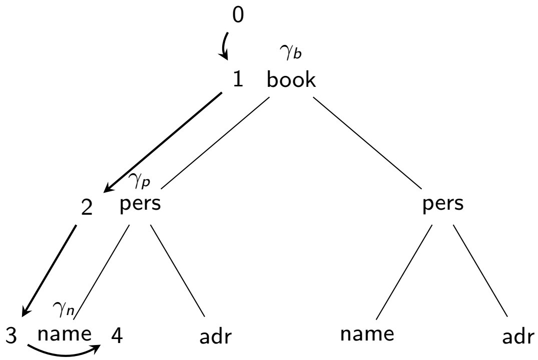
4 &\xrightarrow{\texttt{op}\ \mathit{adr}:\gamma_a} 3\\
3 &\xrightarrow{\texttt{cl}\ \mathit{adr}:\gamma_a} 5\\
5 &\xrightarrow{\texttt{cl}\ \mathit{pers}:\gamma_p} 1\\
1 &\xrightarrow{\texttt{cl}\ \mathit{book}:\gamma_b} 6
\end{aligned}
$$

$< \mathit{book} >$

$< \mathit{pers} >$ $< \mathit{name} >$

Nested Word Automata



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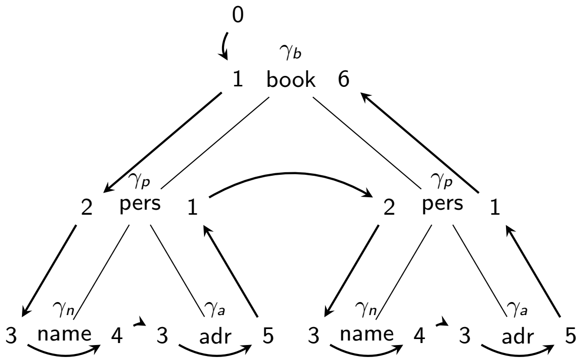
$< book >$

$< pers >$ $< name >$ $< /name >$

Nested Word Automata



$$
\begin{array}{l}
0 \xrightarrow{\texttt{op}\ book:\gamma_b} 1 \\
1 \xrightarrow{\texttt{op}\ pers:\gamma_p} 2 \\
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\end{array}
$$

$< book >$
$< pers > \ < name > \ < /name > \ < adr > < /adr > < /pers >$
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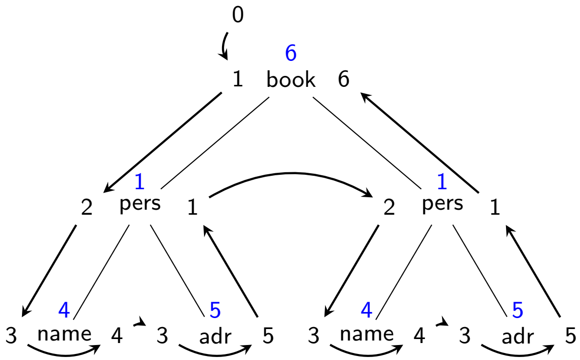
$< /book >$

Nested Word Automata ↓



$$0 \xrightarrow{\texttt{op}\ book:6} 1$$
$$1 \xrightarrow{\texttt{op}\ pers:1} 2$$
$$2 \xrightarrow{\texttt{op}\ name:4} 3$$
$$3 \xrightarrow{\texttt{cl}\ name:4} 4$$
$$4 \xrightarrow{\texttt{op}\ adr:5} 3$$
$$3 \xrightarrow{\texttt{cl}\ adr:5} 5$$
$$5 \xrightarrow{\texttt{cl}\ pers:1} 1$$
$$1 \xrightarrow{\texttt{cl}\ book:6} 6$$

< book >
< pers > < name > < /name > < adr > < /adr > < /pers >
< pers > < name > < /name > < adr > < /adr > < /pers >
< /book >

Nested Word Automata

NA Definition

$T = (\Sigma, \text{states}, \text{stack}, \text{rules}, \text{initial}, \text{final})$

Rules of the form $q \xrightarrow{\texttt{op}\ a:\gamma} q'$ $\quad q \xrightarrow{\texttt{cl}\ a:\gamma} q'$.

Nested Word Automata

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Rules of the form $q \xrightarrow{\texttt{op}\ a:\gamma} q'$ $\qquad q \xrightarrow{\texttt{cl}\ a:\gamma} q'$.

Determinism (dNA)

- In a opening transition $q \xrightarrow{\texttt{op}\ a:\gamma} q'$, q and a determines γ and q'.
- In a closing transition $q \xrightarrow{\texttt{cl}\ a:\gamma} q'$, q, a and γ determines q'.

Nested Word Automata

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- In a opening transition $q \xrightarrow{\texttt{op}\ a:\gamma} q'$, q and a determines γ and q'.
- In a closing transition $q \xrightarrow{\texttt{cl}\ a:\gamma} q'$, q, a and γ determines q'.

Top Down(NA$^{\downarrow}$)

- stack symbols = states
- all closing rules have the form $q \xrightarrow{\texttt{cl}\ a:q'} q'$

Nested Word Transducers

Nested Word to Word (N2W) Definition

$T = (\Sigma, \text{states}, \text{stack}, \text{rules}, \text{initial}, \text{final})$

Rules of the form $q \xrightarrow{\texttt{op}\,a \quad :\gamma} q'$ $\qquad q \xrightarrow{\texttt{cl}\,a \quad :\gamma} q'$

Nested Word Transducers

Nested Word to Word (N2W) Definition

$T = (\Sigma, \Delta, \text{states}, \text{stack}, \text{rules}, \text{initial}, \text{final})$

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Nested Word Transducers

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$[\![T]\!] : \mathcal{T}_\Sigma \to \Delta^*$

Nested Word Transducers

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Determinism and Top Down

Same definition as NA.

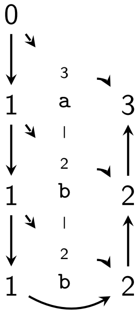
Nested Word Transducer Example

a

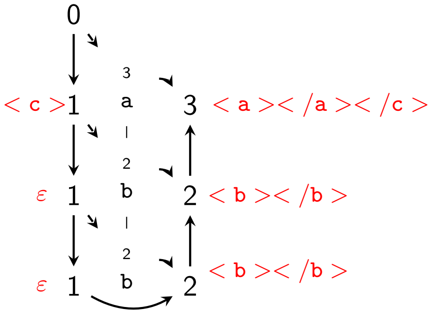
b

b

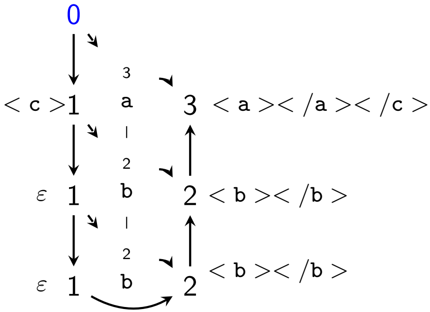
Nested Word Transducer Example



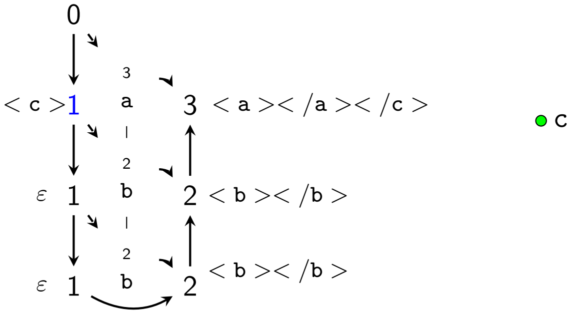
Nested Word Transducer Example



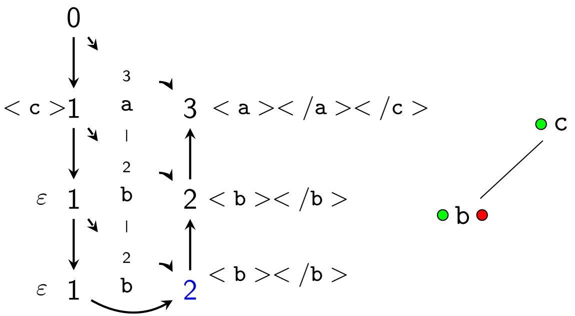
Nested Word Transducer Example



Nested Word Transducer Example



Nested Word Transducer Example



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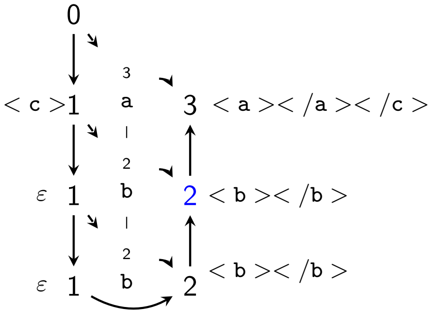
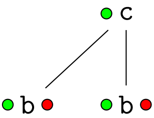
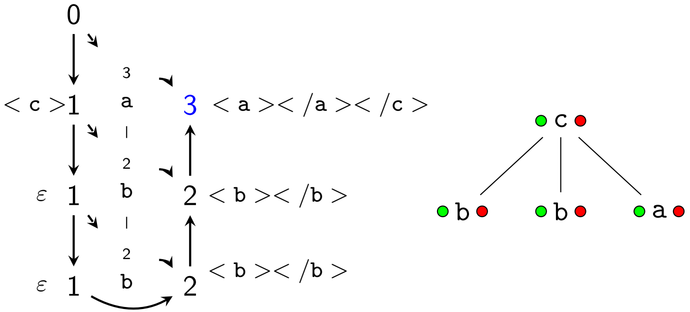




Nested Word Transducer Example



Equivalence Problem

Equivalence of dN2Ws

Two dN2Ws T_1, T_2 are equivalent iff they are defined on the same domain and for each tree t in this domain we have $[\![T_1]\!](t) = [\![T_2]\!](t)$.

- nondeterministic equivalence is undecidable [Griffiths'68]

Outline

Morphism Equivalence on CFG

A (word) *morphism*

- $M : \Sigma \rightarrow \Delta^*$
- $M(v_1 \cdot v_2 \cdots v_n) = M(v_1) \cdot M(v_2) \cdots M(v_n)$

Morphism Equivalence on CFG

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Equivalence on CFG

Two morphisms M_1, M_2 are equivalent on a CFG G iff $M_1(w) = M_2(w)$ for all $w \in L(G)$.

Morphism Equivalence on CFG

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Equivalence on CFG

Two morphisms M_1, M_2 are equivalent on a CFG G iff $M_1(w) = M_2(w)$ for all $w \in L(G)$.

Theorem[Plandowski'94]

The morphism equivalence problem for context-free languages can be solved in polynomial time.

Outline

From Morphisms to $\mathrm{dN2W}^{\downarrow}$

Proposition

Morphism equivalence on CFGs can be reduced in quadratic time to $\mathrm{dN2W}^{\downarrow}$-equivalence.

From Morphisms to dN2W$^{\downarrow}$

Proposition

Morphism equivalence on CFGs can be reduced in quadratic time to dN2W$^{\downarrow}$-equivalence.

Given a CFG G and a morphism M,
we construct a dN2W$^{\downarrow}$ T:

- input : (extended) parse tree t of $w \in L(G)$
- output : $[\![T]\!](t) = M(w)$

From Morphisms to dN2W$^{\downarrow}$

CFG G

$r1$: $S \rightarrow RS$
$r2$: $S \rightarrow R$
$r3$: $R \rightarrow AB$
$r4$: $A \rightarrow a$
$r5$: $B \rightarrow b$

Morphism M

$M(a) = ab$
$M(b) = b$

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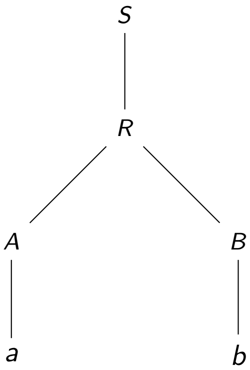

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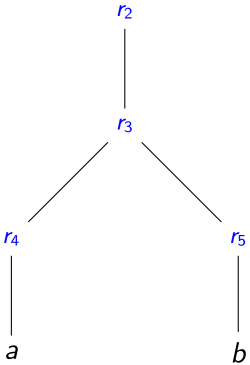

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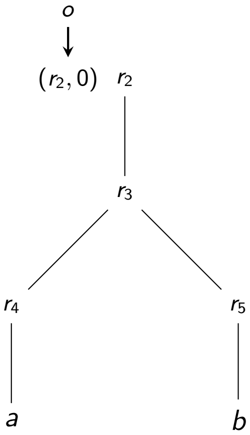

$$o \xrightarrow{r_2:f} (r_2, 0)$$

From Morphisms to dN2W$^{\downarrow}$

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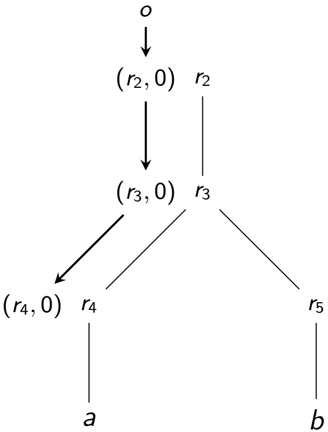

$$(r_2, 0) \xrightarrow{\text{op } r_3:(r_2,1)} (r_3, 0) \qquad (r_3, 0) \xrightarrow{\text{op } r_4:(r_3,1)} (r_4, 0)$$

From Morphisms to dN2W$^{\downarrow}$

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$r1$: $S \rightarrow RS$
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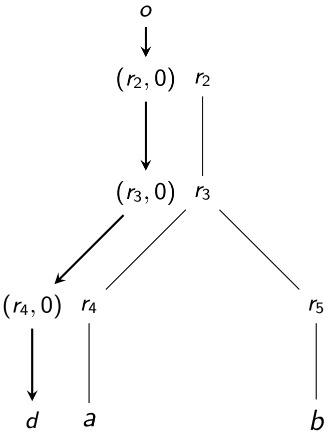

$$(r_4, 0) \xrightarrow{\text{op } a:(r_4,1)} d$$

From Morphisms to $\mathrm{dN2W}^{\downarrow}$

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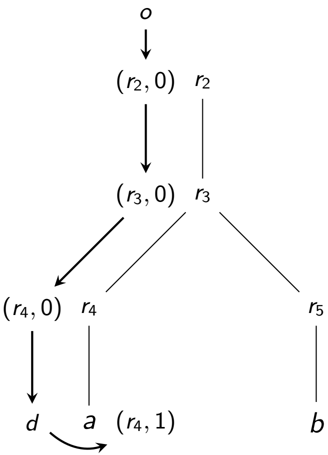

$$d \xrightarrow{\texttt{cl}\ a:(r_4,1)} (r_4, 1)$$

From Morphisms to $\mathrm{dN2W}^{\downarrow}$

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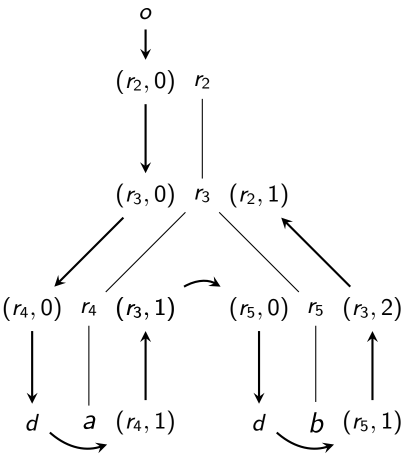

$(r_3, 1) \xrightarrow{\texttt{op}\ r_5:(r_3,2)} (r_5, 0)$ $\qquad$ $(r_5, 1) \xrightarrow{\texttt{cl}\ r_5:(r_3,2)} (r_3, 2)$

From Morphisms to dN2W$^\downarrow$

CFG G

$r1$: $S \rightarrow RS$
$r2$: $S \rightarrow R$
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Morphism M

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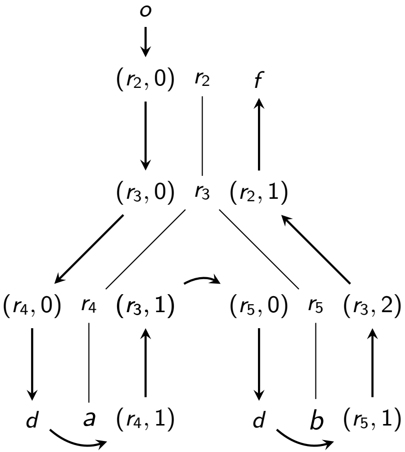


$$(r_2, 1) \xrightarrow{\texttt{cl}\ r_2:f} f$$

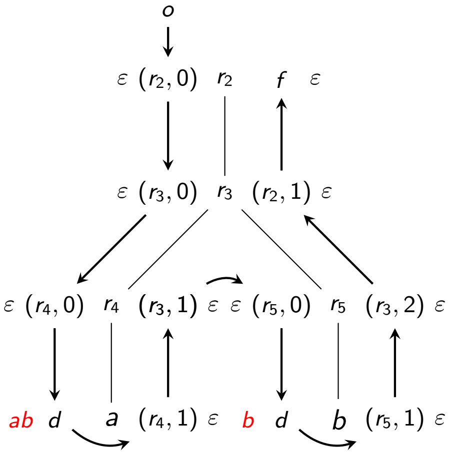
From Morphisms to dN2W$^{\downarrow}$

CFG G

$r1$: $S \to RS$
$r2$: $S \to R$
$r3$: $R \to AB$
$r4$: $A \to a$
$r5$: $B \to b$

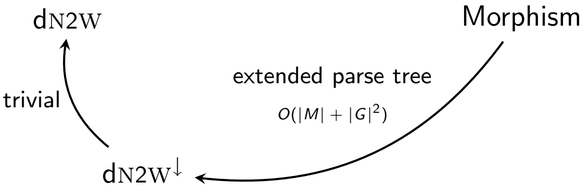
Morphism M

$M(a) = ab$
$M(b) = b$



$(r_4, 0) \xrightarrow{\text{op } a/M(a):(r_4,1)} d \qquad (r_5, 0) \xrightarrow{\text{op } b/M(b):(r_5,1)} d$

From Morphisms to dN2W$^{\downarrow}$



From dN2W to Morphisms

Proposition

dN2W-equivalence can be reduced in polynomial time to morphism equivalence on CFGs.

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dN2W-equivalence can be reduced in polynomial time to morphism equivalence on CFGs.

For dN2W T_1 and T_2, we define :

- a CFG G which recognizes successfull parallel runs, $L(G) \subseteq (\text{rules}_{T_1} \times \text{rules}_{T_2})^*$.
- two morphisms M_1 and M_2 s.t.: For all $s \in L(G)$, $M_1(s) = [\![T_1]\!](t)$ and $M_2(s) = [\![T_2]\!](t)$.

From dN2W to Morphisms

T_1

$(init = \{0\}, fin = \{3\})$

r_1: $0 \xrightarrow{\texttt{op}\ a/{<}c{>}:\gamma_1} 1$

r_2: $1 \xrightarrow{\texttt{op}\ b/\varepsilon:\gamma_2} 1$

r_3: $1 \xrightarrow{\texttt{cl}\ b/{<}b{>}{<}/b{>}:\gamma_2} 2$

r_4: $2 \xrightarrow{\texttt{cl}\ a/{<}a{>}{<}/a{>}{<}/c{>}:\gamma_1} 3$

T_2

$(init = \{0'\}, fin = \{4'\})$

r'_1: $0' \xrightarrow{\texttt{op}\ a/{<}c{>}:\gamma'_1} 1'$

r'_2: $1' \xrightarrow{\texttt{op}\ b/{<}b{>}:\gamma'_2} 2'$

r'_3: $3' \xrightarrow{\texttt{cl}\ a/{<}/a{>}{<}/c{>}:\gamma'_1} 4'$

r'_4: $2' \xrightarrow{\texttt{cl}\ b/{<}/b{>}{<}a{>}:\gamma'_2} 3'$

From dN2W to Morphisms

T_1

$(init = \{0\}, fin = \{3\})$

r_1: $0 \xrightarrow{\texttt{op}\ a/{<}c{>}:\gamma_1} 1$

r_2: $1 \xrightarrow{\texttt{op}\ b/\varepsilon:\gamma_2} 1$

r_3: $1 \xrightarrow{\texttt{cl}\ b/{<}b{>}{<}/b{>}:\gamma_2} 2$

r_4: $2 \xrightarrow{\texttt{cl}\ a/{<}a{>}{<}/a{>}{<}/c{>}:\gamma_1} 3$

T_2

$(init = \{0'\}, fin = \{4'\})$

r'_1: $0' \xrightarrow{\texttt{op}\ a/{<}c{>}:\gamma'_1} 1'$

r'_2: $1' \xrightarrow{\texttt{op}\ b/{<}b{>}:\gamma'_2} 2'$

r'_3: $3' \xrightarrow{\texttt{cl}\ a/{<}/a{>}{<}/c{>}:\gamma'_1} 4'$

r'_4: $2' \xrightarrow{\texttt{cl}\ b/{<}/b{>}{<}a{>}:\gamma'_2} 3'$

$$\begin{tikzcd}
(r_1, r'_1) \arrow[d] & \texttt{a} \arrow[d, dash] & (r_4, r'_3) \\
(r_2, r'_2) & \texttt{b} \arrow[loop below] & (r_3, r'_4) \arrow[u]
\end{tikzcd}$$

From dN2W to Morphisms

T_1

$(init = \{0\}, fin = \{3\})$

r_1: $0 \xrightarrow{\texttt{op}\ a/<c>:\gamma_1} 1$

r_2: $1 \xrightarrow{\texttt{op}\ b/\varepsilon:\gamma_2} 1$

r_3: $1 \xrightarrow{\texttt{cl}\ b/:\gamma_2} 2$

r_4: $2 \xrightarrow{\texttt{cl}\ a/<a></c>:\gamma_1} 3$

T_2

$(init = \{0'\}, fin = \{4'\})$

r'_1: $0' \xrightarrow{\texttt{op}\ a/<c>:\gamma'_1} 1'$

r'_2: $1' \xrightarrow{\texttt{op}\ b/:\gamma'_2} 2'$

r'_3: $3' \xrightarrow{\texttt{cl}\ a/</c>:\gamma'_1} 4'$

r'_4: $2' \xrightarrow{\texttt{cl}\ b/<a>:\gamma'_2} 3'$

$$\begin{tikzcd} (r_1, r'_1) \arrow[d] & \mathtt{a} \arrow[d, dash] & (r_4, r'_3) \\ (r_2, r'_2) & \mathtt{b} & (r_3, r'_4) \arrow[u] \end{tikzcd}$$

$$[\mathbf{S}] \rightarrow (r_1, r'_1) \cdot [(\mathbf{1}, \mathbf{2}), (\mathbf{1}', \mathbf{3}')] \cdot (r_4, r'_3)$$

From dN2W to Morphisms

T_1

$(init = \{0\}, fin = \{3\})$

r_1: $0 \xrightarrow{\texttt{op}\ a/<c>:\gamma_1} 1$

r_2: $1 \xrightarrow{\texttt{op}\ b/\varepsilon:\gamma_2} 1$

r_3: $1 \xrightarrow{\texttt{cl}\ b/:\gamma_2} 2$

r_4: $2 \xrightarrow{\texttt{cl}\ a/<a></c>:\gamma_1} 3$

T_2

$(init = \{0'\}, fin = \{4'\})$

r'_1: $0' \xrightarrow{\texttt{op}\ a/<c>:\gamma'_1} 1'$

r'_2: $1' \xrightarrow{\texttt{op}\ b/:\gamma'_2} 2'$

r'_3: $3' \xrightarrow{\texttt{cl}\ a/</c>:\gamma'_1} 4'$

r'_4: $2' \xrightarrow{\texttt{cl}\ b/<a>:\gamma'_2} 3'$

$$\begin{tikzcd}
(r_1, r'_1) \arrow[d] & \mathtt{a} \arrow[d, dash] & (r_4, r'_3) \\
(r_2, r'_2) & \mathtt{b} & (r_3, r'_4) \arrow[u]
\end{tikzcd}$$

$$[\mathbf{S}] \to (r_1, r'_1) \cdot [(\mathbf{1}, \mathbf{2}), (\mathbf{1}', \mathbf{3}')] \cdot (r_4, r'_3)$$

$$[(\mathbf{1}, \mathbf{2}), (\mathbf{1}', \mathbf{3}')] \to (r_2, r'_2) \cdot [(\mathbf{1}, \mathbf{1}), (\mathbf{2}', \mathbf{2}')] \cdot (r_3, r'_4)$$

From dN2W to Morphisms

T_1

$(init = \{0\}, fin = \{3\})$

r_1: $0 \xrightarrow{\mathtt{op}\ a/{<}c{>}:\gamma_1} 1$

r_2: $1 \xrightarrow{\mathtt{op}\ b/\varepsilon:\gamma_2} 1$

r_3: $1 \xrightarrow{\mathtt{cl}\ b/{<}b{>}{<}/b{>}:\gamma_2} 2$

r_4: $2 \xrightarrow{\mathtt{cl}\ a/{<}a{>}{<}/a{>}{<}/c{>}:\gamma_1} 3$

T_2

$(init = \{0'\}, fin = \{4'\})$

r'_1: $0' \xrightarrow{\mathtt{op}\ a/{<}c{>}:\gamma'_1} 1'$

r'_2: $1' \xrightarrow{\mathtt{op}\ b/{<}b{>}:\gamma'_2} 2'$

r'_3: $3' \xrightarrow{\mathtt{cl}\ a/{<}/a{>}{<}/c{>}:\gamma'_1} 4'$

r'_4: $2' \xrightarrow{\mathtt{cl}\ b/{<}/b{>}{<}a{>}:\gamma'_2} 3'$

$$\begin{tikzcd} (r_1, r'_1) \arrow[d] & \mathtt{a} \arrow[d, dash] & (r_4, r'_3) \\ (r_2, r'_2) & \mathtt{b} & (r_3, r'_4) \arrow[u] \end{tikzcd}$$

$$[\mathbf{S}] \rightarrow (r_1, r'_1) \cdot [(\mathbf{1},\mathbf{2}),(\mathbf{1}',\mathbf{3}')] \cdot (r_4, r'_3)$$
$$[(\mathbf{1},\mathbf{2}),(\mathbf{1}',\mathbf{3}')] \rightarrow (r_2, r'_2) \cdot [(\mathbf{1},\mathbf{1}),(\mathbf{2}',\mathbf{2}')] \cdot (r_3, r'_4)$$
$$[(\mathbf{1},\mathbf{1}),(\mathbf{2}',\mathbf{2}')] \rightarrow \varepsilon$$

From dN2W to Morphisms

T_1

$(init = \{0\}, fin = \{3\})$

r_1: $0 \xrightarrow{\texttt{op}\ a/<c>:\gamma_1} 1$

r_2: $1 \xrightarrow{\texttt{op}\ b/\varepsilon:\gamma_2} 1$

r_3: $1 \xrightarrow{\texttt{cl}\ b/:\gamma_2} 2$

r_4: $2 \xrightarrow{\texttt{cl}\ a/<a></c>:\gamma_1} 3$

T_2

$(init = \{0'\}, fin = \{4'\})$

r'_1: $0' \xrightarrow{\texttt{op}\ a/<c>:\gamma'_1} 1'$

r'_2: $1' \xrightarrow{\texttt{op}\ b/:\gamma'_2} 2'$

r'_3: $3' \xrightarrow{\texttt{cl}\ a/</c>:\gamma'_1} 4'$

r'_4: $2' \xrightarrow{\texttt{cl}\ b/<a>:\gamma'_2} 3'$

$$\begin{tikzcd}
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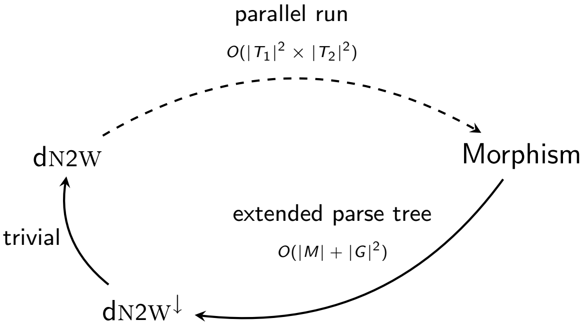
\end{tikzcd}$$

$$[\mathbf{S}] \to (r_1, r'_1) \cdot [(\mathbf{1}, \mathbf{2}), (\mathbf{1}', \mathbf{3}')] \cdot (r_4, r'_3)$$
$$[(\mathbf{1}, \mathbf{2}), (\mathbf{1}', \mathbf{3}')] \to (r_2, r'_2) \cdot [(\mathbf{1}, \mathbf{1}), (\mathbf{2}', \mathbf{2}')] \cdot (r_3, r'_4)$$
$$[(\mathbf{1}, \mathbf{1}), (\mathbf{2}', \mathbf{2}')] \to \varepsilon$$

$M_1((r_3, r'_4)) =< b >< /b >$ $\quad$ $M_2((r_3, r'_4)) =< /b >< a >$

From dN2W to Morphisms



Other Models

Top Down Ranked Tree to Word ($\mathrm{dR2W}^{\downarrow}$)

$$q(a(\mathtt{x}_1, \ldots, \mathtt{x}_k)) \to u_0 \cdot q_1(\mathtt{x}_1) \cdot u_1 \cdot \ldots \cdot u_{k-1} \cdot q_k(\mathtt{x}_k) \cdot u_k.$$

Other Models

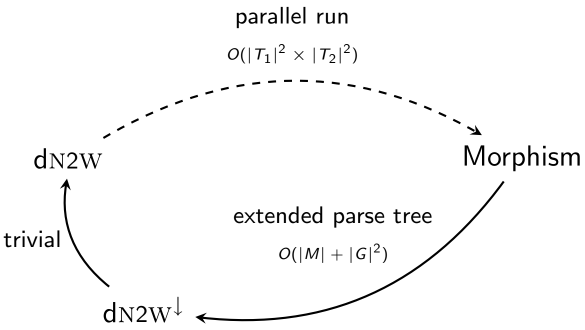
Top Down Ranked Tree to Word ($\mathrm{dR2W}^{\downarrow}$)

$$q(a(\mathtt{x}_1, \ldots, \mathtt{x}_k)) \to u_0 \cdot q_1(\mathtt{x}_1) \cdot u_1 \cdot \ldots \cdot u_{k-1} \cdot q_k(\mathtt{x}_k) \cdot u_k.$$

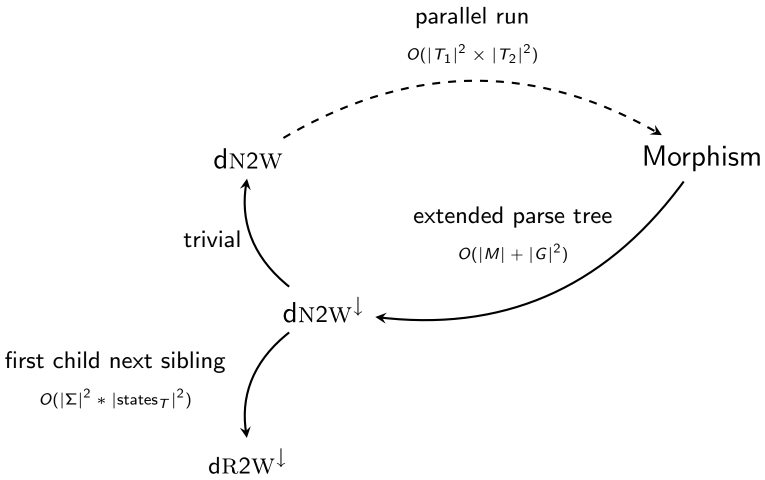
Deterministic Bottom Up Ranked Tree to Word ($\mathrm{dR2W}^{\uparrow}$)

$$a(q_1(v_1), \ldots, q_k(v_k)) \to q(u_0 \cdot v_1 \cdot u_1 \cdots v_k \cdot u_k).$$

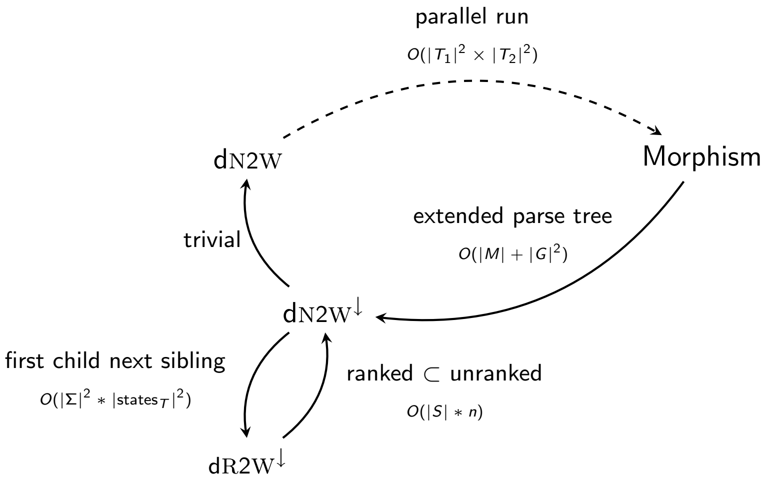
Other Models



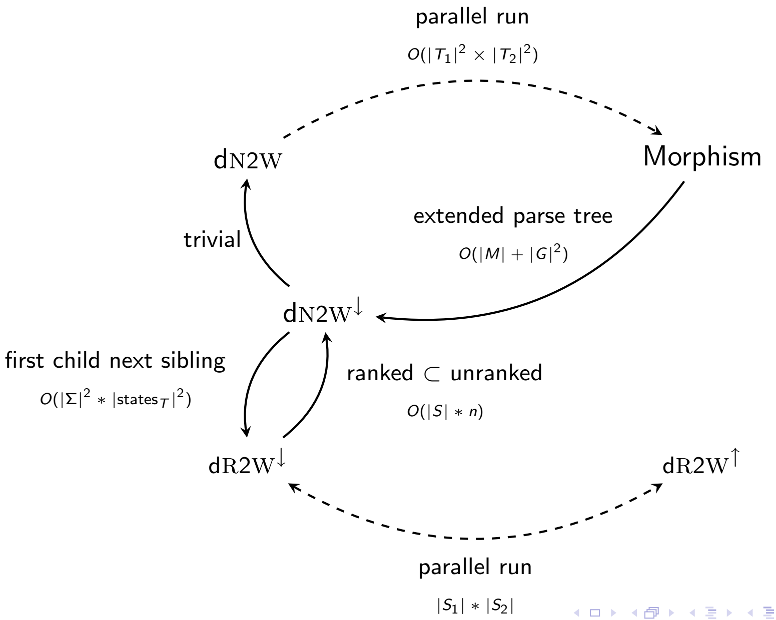
Other Models



Other Models



Other Models



Outline

Conclusion

Summary

- new model of deterministic nested word transducers.
- equivalence problem on various classes.
- relation to morphisms equivalence on CFG.

Future work

- other problems on dN2W.
- grammatical inference.

From Morphisms to $\mathrm{dN2W}^{\downarrow}$

$$\frac{r \in \mathrm{rules}_G \quad lhs(r) \in \mathrm{initial}_G}{\begin{array}{c} \mathtt{o} \xrightarrow{\mathtt{op}\ r/\varepsilon:\mathtt{f}} (r,0) \\ (r,|r|) \xrightarrow{\mathtt{cl}\ r/\varepsilon:\mathtt{f}} \mathtt{f} \end{array}} \qquad \frac{r \in \mathrm{rules}_G \quad rhs(r) = a}{\begin{array}{c} (r,0) \xrightarrow{\mathtt{op}\ a/M(a):(r,1)} \mathtt{d} \\ \mathtt{d} \xrightarrow{\mathtt{cl}\ a/\varepsilon:(r,1)} (r,1) \end{array}}$$

$$\frac{r, r' \in \mathrm{rules}_G \quad rhs(r) = q_1 \cdots q_k \quad 1 \leq j \leq |r| \quad lhs(r') = q_j}{\begin{array}{c} (r, j{-}1) \xrightarrow{\mathtt{op}\ r'/\varepsilon:(r,j)} (r',0) \\ (r',|r'|) \xrightarrow{\mathtt{cl}\ r'/\varepsilon:(r,j)} (r,j) \end{array}}$$

From dN2W to Morphisms

$$\frac{r_1, r_1' \in \text{rules}_{T_1} \quad r_1 = p_1 \xrightarrow{\texttt{op}\ a/u_1:\gamma_1} q_1 \quad r_1' = p_1' \xrightarrow{\texttt{cl}\ a/u_1':\gamma_1} q_1' \quad p_1 \in \text{initial}_{T_1} \quad q_1' \in \text{final}_{T_1} \atop r_2, r_2' \in \text{rules}_{T_2} \quad r_2 = p_2 \xrightarrow{\texttt{op}\ a/u_2:\gamma_2} q_2 \quad r_2' = p_2' \xrightarrow{\texttt{cl}\ a/u_2':\gamma_2} q_2' \quad p_2 \in \text{initial}_{T_2} \quad q_2' \in \text{final}_{T_2}}{\text{o} \to (r_1, r_2) \cdot ((q_1, p_1'), (q_2, p_2')) \cdot (r_1', r_2')}$$

$$\frac{r_1, r_1' \in \text{rules}_{T_1} \quad r_1 = p_1 \xrightarrow{\texttt{op}\ a/u_1:\gamma_1} q_1 \quad r_1' = p_1' \xrightarrow{\texttt{cl}\ a/u_1':\gamma_1} q_1', \atop r_2, r_2' \in \text{rules}_{T_2} \quad r_2 = p_2 \xrightarrow{\texttt{op}\ a/u_2:\gamma_2} q_2 \quad r_2' = p_2' \xrightarrow{\texttt{cl}\ a/u_2':\gamma_2} q_2'}{((p_1, q_1'), (p_2, q_2')) \to (r_1, r_2) \cdot ((q_1, p_1'), (q_2, p_2')) \cdot (r_1', r_2')}$$

$$\frac{p_1, p_1', q_1 \in \text{states}_{T_1} \quad p_2, p_2', q_2 \in \text{states}_{T_2}}{((p_1, q_1), (p_2, q_2)) \to ((p_1, p_1'), (p_2, p_2')) \cdot ((p_1', q_1), (p_2', q_2))} \qquad \frac{q_1 \in \text{states}_{T_1},\ q_2 \in \text{states}_{T_2}}{((q_1, q_1), (q_2, q_2)) \to \varepsilon}$$